

Arithmetic of negative numbers

These notes attempt to give you some intuition and understanding about the rules for the arithmetic of negative numbers, by using the example of money being paid into and out of a bank account.

A bank account that is in credit has a positive balance, such as +£10.

A bank account that is in debit – that is, where the account holder owes money to the bank – has a negative balance, such as –£20.

The discussion below presents a series of examples, each of which illustrates a rule for the arithmetic of negative numbers. The rules are given in boxes following the examples. The discussion starts by considering addition and subtraction, and then looks at multiplication and division.

Addition and subtraction

Adding a positive number

This occurs when you pay money into the account.

Adding £5 to an account with £3 in it results in a balance of +£8.

$$3 + 5 = 8.$$

Adding £3 to an account with –£6 in it results in a balance of –£3:

$$-6 + 3 = -3.$$

To add a positive number, you increase the number that you started with by the positive number.

Subtracting a positive number

This occurs when you make a payment from the account.

Taking £3 from an account with £3 in it results in a balance of £0:

$$3 - 3 = 0.$$

Taking £3 from an account with £0 in it results in a balance of –£3 (a debt of £3):

$$0 - 3 = -3.$$

Taking £3 from an account with –£6 in it results in a balance of –£9 (a debt of –£9):

$$-6 - 3 = -9.$$

To subtract a positive number, you decrease the number that you started with by the positive number.

Adding a negative number

This occurs when a debt is added to the account; for example when the bank adds a charge for some service. The charge can be represented by a negative number, and adding it to the account reduces the amount of money in the account.

Adding a bank charge of $-\pounds 3$ to an account with $\pounds 6$ in it results in a balance of $+\pounds 3$:

$$6 + (-3) = 3.$$

Adding a bank charge of $-\pounds 3$ to an account with $\pounds 3$ in it results in a balance of $\pounds 0$:

$$3 + (-3) = 0.$$

Adding a bank charge of $-\pounds 3$ to an account with $\pounds 0$ in it results in a balance of $-\pounds 3$:

$$0 + (-3) = -3.$$

Adding a negative number is the same as subtracting the corresponding positive number.

Subtracting a negative number

This occurs when a debt, represented by a negative number, is taken away, perhaps by the bank agreeing to cancel some or all of your debt (that would be nice!) or by your rich uncle agreeing to wipe out your debt.

Removing $-\pounds 3$ (a debt of $\pounds 3$) from an account with $-\pounds 3$ in it results in a balance of $\pounds 0$:

$$(-3) - (-3) = 0.$$

Removing $-\pounds 3$ (a debt of $\pounds 3$) from an account with $-\pounds 6$ in it results in a balance of $-\pounds 3$ (the account holder is still in debt to the bank):

$$(-6) - (-3) = -3.$$

What your rich uncle might actually do to cancel some or all of your debt is to pay some money into your account. So removing a debt has the same effect as adding some money to the account.

For example, adding $\pounds 3$ to an account with $-\pounds 3$ in it (a debt of $\pounds 3$) results in a balance of $\pounds 0$:

$$(-3) + 3 = 0.$$

This has the same effect as cancelling a debt of $-\pounds 3$:

$$(-3) - (-3) = (-3) + 3 = 0.$$

Subtracting a negative number is the same as adding the corresponding positive number.

Multiplication and division

Multiplying a positive number by a positive number

Suppose that £3 is added to your account every month for 12 months. Just as adding £3 once results in a change of +£3, so adding £3 twelve times results in a change of

$$12 \times £3 = +£36.$$

A positive number times a positive number is a positive number.

Multiplying a negative number by a positive number

Suppose that you remove £3 from your account every month for 12 months. Just as removing £3 once results in a change of

$$(-1) \times £3 = -£3,$$

so removing £3 twelve times results in a change of

$$(-12) \times £3 = -£36.$$

A negative number times a positive number is a negative number.

Multiplying a negative number by a negative number

Suppose that your rich uncle agrees to remove -£3 (a debt of £3) from your account every month for 12 months. Just as removing -£3 once results in a change of

$$(-1) \times (-£3) = +£3,$$

so removing -£3 twelve times results in a change of

$$(-12) \times (-£3) = +£36.$$

A negative number times a negative number is a positive number.

Division

The rules for the sign of the answer when you divide one positive or negative number by another are the same as the rules for when you multiply two numbers, and are not discussed in detail here.